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MODELING THE BEHAVIOR OF THE PHYSICAL AND GEOMETRIC NON-LINEAR FUNCTIONAL HETEROGENEOUS MATERIALS

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The work is devoted to the problem of modeling the behavior of functionally inhomogeneous materials with the properties of pseudo-elastic-plasticity under complex loads, in particular at large strains (up to 20%), when geometric nonlinearity in Cauchy relations must be taken into account. In previous works of the authors, functionally heterogeneous materials were studied in a geometrically linear formulation, which is true for small deformations (up to 7%). When predicting work with material at large deformations, it is necessary to take into account geometric nonlinearity in Cauchy relations.

Studying the behavior of bodies made of functionally heterogeneous materials under unsteady load requires the development of special approaches, methods and algorithms for calculating the stress-strain state. When constructing physical relations, it is assumed that the deformation at the point is represented as the sum of the elastic component, the jump in deformation during the phase transition, plastic deformation and deformation caused by temperature changes.

A physical relationship in a nonlinear setting is proposed for modeling the behavior of bodies made of functionally heterogeneous materials. Formulas are obtained that nonlinearly relate strain rates and Formulas are obtained that nonlinearly relate strain rates and displacement rates.

Keywords: mathematical modeling, functional heterogeneous materials, geometric nonlinearity, spline functions, pseudo-elastic plasticity, phase transitions

Problem statement. The main task of the non-stationary theory of thermo-plasticity is to determine the rates of displacement and components of stress and strain tensors that occur in a three-dimensional body during its loading and heating, when some elements of the body work beyond the elasticity of the material. The loading process will be considered as one that changes over time, which can cause the movement of individual parts of the body.

At first, an isotropic and homogeneous three-dimensional body V, bounded by the surface S, at the initial moment of time $t = 0$ is in a natural non-stressed state at the temperature $T_o(\theta_i)$, $i = 1, 2, 3$. Then the body is subjected to heat and load by external forces. These can be volumetric forces that affect each element of the body. Surface forces acting on one part of the body's surface.

On the second part of the body's surface, which can be fixed in a certain way, the speed of movement is set as a function of coordinates and time. Let's assume that the heating and loading of the body occur in such a way that there are deformations that can significantly affect the temperature change of this element. We will consider such loading processes and temperature levels at which the rheological properties of the material are not detected. The configuration of a body is given by the equation of the surface bounding it. In addition, you need to set the thermo-physical and mechanical characteristics of the body material and the conditions for its heat exchange with the environment.

The thermo-physical properties of the material are characterized by thermal and thermal conductivity coefficients, which may depend on the temperature. The heat exchange conditions are set in the form of corresponding boundary conditions, and the mechanical characteristics of the material in the study of deformation processes along rectilinear paths and the trajectory of small curvature are set in the form of instantaneous stretching diagrams of samples obtained at various fixed temperatures. In addition, the values of Poisson's coefficients ν and linear thermal expansion are set.

Based on these data, it is necessary to determine the temperature, three components of the displacement velocity vector, six components of the stress tensor and six components of the strain tensor. So we need to determine sixteen unknown functions of time and three coordinates. To do this, you need to use the equations of motion, geometric and physical equations, as well as the equation of thermal conductivity. The temperature field at an arbitrary point of the body in the presence of heat sources in it and in the case of accounting for the heat that is released during its deformation, is determined by solving the thermal conductivity equation under certain initial and boundary conditions [4, 5].

After determining the temperature field for various moments of time, the components of the velocity vector of displacement and the components of stress and strain tensors that satisfy three differential equations of motion, six geometric equations, and six physical equations are searched for. These equations are solved under certain initial and boundary conditions. Initial conditions are set for all unknowns at the initial time. On the part of the surface of the body where the given forces *b(xi, t),* the components of the stress tensor must satisfy three boundary conditions:

$\sigma_{in}\left(\alpha_k, t\right) = \sigma_{ij} \cdot n_j, i = 1,2,3,$ where n_j is the guiding cosines of the external normal to the surface of the body at the corresponding point. On the rest of the surface, where the components of the displacement velocity vector are set, the displacement velocities must take the specified values:

$$v_i = V_i\left(x_j, t\right).$$

A different formulation of boundary conditions is possible, when three conditions are set on the surface of the body, taken in a certain way from the above conditions.

We will find the definition of unknowns as follows. Three components of the displacement velocity vector and six components of the stress tensor are taken as the main unknowns, for which boundary conditions are directly formulated. In this case, all components of the strain tensor are excluded from the six physical equations using geometrically nonlinear Cauchy relations, which are then determined based on the already known components of the displacement velocity vector.

When solving the non-stationary problem of thermo-plasticity, we will use the defining equations that describe non-isothermal load processes both along rectilinear trajectories and along the trajectories of small curvature deformation. After completing the task on the geometry of the deformation trajectory, you can conclude that the determining relationships used are reliable.

Solving the problem.

One aspect of the numerical solution of general nonstationary problems for inelastic bodies is the choice of the physical relationship between stress and strain. This choice is consistent with experiments and is closely related to the deformation processes occurring in the body material. In the general case, the magnitudes of deformation are functions of the process of stresses and temperature differences, which are determined by the characteristics of the entire previous process of changes in physical factors, and not just the current values. Detailed information on this issue can be found in [2].

When constructing physical relations, it was assumed that the deformation at a point is represented as the sum of the elastic component, the deformation jump at the phase transition, plastic deformation and deformation caused by temperature differences [dissertation].

The equations of motion of an infinitesimal volume element of a continuous medium that is deformed in the orthogonal coordinate system α^1, α^2, α^3 in are represented as:

$$\frac{\partial v_i}{\partial t} = \frac{1}{\rho} \cdot \frac{\partial \sigma_{ij}}{\partial \alpha^j} + B_i\left(\sigma_{mn}\right), \qquad (1)$$

where *i, j, n, m* =1, 2, 3, and ρ - density. Magnitude $B_i(\sigma_{mn}),$, that are in the right side of equation (1) given in [5,7], and: $v_i = \frac{\partial u_i}{\partial t},\ i = 1,2,3.$

In general, the System of orthogonal coordinate of the strain tensor and the components of the displacement vector are connected by such nonlinear relations [6]:

$$\varepsilon_{11} = e_{11} + \frac{1}{2}\left[e_{11}^2 + \left(\frac{e_{12}}{2} + \omega_3\right)^2 + \left(\frac{e_{13}}{2} - \omega_2\right)^2 \right]; \quad \begin{aligned} \varepsilon_{12} = e_{12} + e_{11}\left(\frac{e_{12}}{2} - \omega_3\right) + \\ + e_{22}\left(\frac{e_{12}}{2} + \omega_3\right) + \\ + \left(\frac{e_{13}}{2} - \omega_2\right)\left(\frac{e_{23}}{2} + \omega_1\right); \end{aligned} \qquad (2)$$

Other components of the strain tensor are obtained from (1) by cyclic index permutation. In the case of an orthogonal rectangular coordinate system:

$$\begin{aligned} & e_{11} = \frac{\partial u_1}{\partial \alpha_1}; e_{22} = \frac{\partial u_2}{\partial \alpha_2};\ e_{12} = \frac{\partial u_2}{\partial \alpha_1} + \frac{\partial u_1}{\partial \alpha_2}; e_{23} = \frac{\partial u_3}{\partial \alpha_2} + \frac{\partial u_2}{\partial \alpha_3}; \\ & e_{33} = \frac{\partial u_3}{\partial \alpha_3}; \qquad e_{31} = \frac{\partial u_1}{\partial \alpha_3} + \frac{\partial u_3}{\partial \alpha_1}; \end{aligned} \qquad (3)$$

$$2\omega_1 = \frac{\partial u_3}{\partial \alpha_2} - \frac{\partial u_2}{\partial \alpha_3}; 2\omega_2 = \frac{\partial u_1}{\partial \alpha_3} - \frac{\partial u_3}{\partial \alpha_1}; 2\omega_3 = \frac{\partial u_2}{\partial \alpha_1} - \frac{\partial u_1}{\partial \alpha_2};$$

With this in mind, after time differentiation in the geometrically nonlinear case, it is possible to record the following for strain rates:

$$\frac{\partial \varepsilon_{11}}{\partial t} = \left(1+e_{11}\right)\frac{\partial v_1}{\partial \alpha_1} + \left(\frac{e_{12}}{2}+\omega_3\right)\frac{\partial v_2}{\partial \alpha_1} + \left(\frac{e_{13}}{2}-\omega_2\right)\frac{\partial v_3}{\partial \alpha_1};$$

$$\frac{\partial \varepsilon_{22}}{\partial t} = \left(\frac{e_{21}}{2}-\omega_3\right)\frac{\partial v_1}{\partial \alpha_2} + \left(1+e_{22}\right)\frac{\partial v_2}{\partial \alpha_2} + \left(\frac{e_{23}}{2}+\omega_1\right)\frac{\partial v_3}{\partial \alpha_2};$$

$$\frac{\partial \varepsilon_{33}}{\partial t} = \left(\frac{e_{31}}{2}+\omega_2\right)\frac{\partial v_1}{\partial \alpha_3} + \left(\frac{e_{32}}{2}-\omega_1\right)\frac{\partial v_2}{\partial \alpha_3} + \left(1+e_{33}\right)\frac{\partial v_3}{\partial \alpha_3};$$

$$\frac{\partial \varepsilon_{12}}{\partial t} = \left(1+\frac{e_{11}}{2}+\frac{e_{22}}{2}\right)\left(\frac{\partial v_2}{\partial \alpha_1}+\frac{\partial v_1}{\partial \alpha_2}\right) + \left(\frac{e_{12}}{2}-\omega_3\right)\frac{\partial v_1}{\partial \alpha_1} + \left(\frac{e_{12}}{2}+\omega_3\right)\frac{\partial v_2}{\partial \alpha_2} +$$

$$+\frac{\left(e_{22}-e_{11}\right)}{2}\left(\frac{\partial v_2}{\partial \alpha_1}-\frac{\partial v_1}{\partial \alpha_2}\right) + \left(\frac{e_{23}}{2}+\omega_1\right)\frac{\partial v_3}{\partial \alpha_1} + \left(\frac{e_{13}}{2}-\omega_2\right)\frac{\partial v_3}{\partial \alpha_2};$$

$$\frac{\partial \varepsilon_{23}}{\partial t} = \left(1+\frac{e_{22}}{2}+\frac{e_{33}}{2}\right)\left(\frac{\partial v_3}{\partial \alpha_2}+\frac{\partial v_2}{\partial \alpha_3}\right) + \left(\frac{e_{23}}{2}-\omega_1\right)\frac{\partial v_2}{\partial \alpha_2} + \left(\frac{e_{23}}{2}+\omega_1\right)\frac{\partial v_3}{\partial \alpha_3} +$$

$$+\frac{\left(e_{33}-e_{22}\right)}{2}\left(\frac{\partial v_3}{\partial \alpha_2}-\frac{\partial v_2}{\partial \alpha_3}\right) + \left(\frac{e_{31}}{2}+\omega_2\right)\frac{\partial v_1}{\partial \alpha_2} + \left(\frac{e_{21}}{2}-\omega_3\right)\frac{\partial v_1}{\partial \alpha_3}; \qquad (4)$$

$$\frac{\partial \varepsilon_{31}}{\partial t} = \left(1+\frac{e_{33}}{2}+\frac{e_{11}}{2}\right)\left(\frac{\partial v_1}{\partial \alpha_3}+\frac{\partial v_3}{\partial \alpha_1}\right) + \left(\frac{e_{31}}{2}-\omega_2\right)\frac{\partial v_3}{\partial \alpha_3} + \left(\frac{e_{31}}{2}+\omega_2\right)\frac{\partial v_1}{\partial \alpha_1} +$$

$$+\frac{\left(e_{11}-e_{33}\right)}{2}\left(\frac{\partial v_1}{\partial \alpha_3}-\frac{\partial v_3}{\partial \alpha_1}\right) + \left(\frac{e_{12}}{2}+\omega_3\right)\frac{\partial v_2}{\partial \alpha_3} + \left(\frac{e_{32}}{2}-\omega_1\right)\frac{\partial v_2}{\partial \alpha_1},$$

The system of equations (1), (4) is closed by physical relations that connect stresses and deformations.

Let us determine the influence of the shape memory of the material on the behavior of the rod. Consider the problem of loading, subsequent unloading of a one-dimensional rod and its subsequent heating, which simulates the influence of the memory of the shape of the material at a material point on the behavior of the rod as a whole.

In the first stage, the solution of the previous problem was obtained. For the time interval on the edge, the speed at which the sample is stretched is set. After that the condition for the edge is fulfilled.

The edge is fixed and here the speed of movement is zero. The distribution of strain and stress for is presented in Table 1. The deformation has an almost constant value along its entire length. We use these results as initial conditions in the second stage.

The numerical results of the application of the above approach using formulas (1) - (4) for the geometrically linear and nonlinear case are as follows:

Table 1

Distribution of deformation and stress before heating

$\varepsilon_{лін}$	4.99	5.00	4.995	4.992	4.998	5.00	4.998	5.01	5.00	4.998	5.00	5.003	4.997
$\varepsilon_{нелін}$	5.00	4.99	4.995	4.997	4.998	4.997	4.996	4.997	4.992	4.998	4.994	4.994	4.997

In the second stage, the rod is heated evenly along the length under constant boundary conditions. The distribution of strain and stress for the respective moments of time and temperature are presented in tables 2 and 3.

Table 2

Distribution along the length of the rod deformation and stress when heated for different times and temperatures 40 ^{0}C

ε	0.18	0.12	0.09	0.26	0.36	0.22	0.35	0.28	0.41	0.20	0.22	0.44
σ	0.0	0.1	0.07	0.26	0.33	0.20	0.32	0.26	0.38	0.18	0.19	0.40

Table 3

Distribution along the length of the rod of deformation and stress when heated for different times and temperatures 70 ^{0}C

ε	0.00	0.08	0.02	0.2	0.8	0.6	0.00	0.15	0.18	0.14	0.05	0.02
σ	0.00	0.18	0.14	0.1	0.1	0.2	0.00	0.05	0.06	0.03	0.02	0.01

Conclusions: The paper proposes physical relations in a nonlinear formulation for modeling the behavior of bodies from functionally inhomogeneous materials. Formulas are obtained that nonlinearly relate strain rates and displacement rates. Nonlinear geometric relations make it possible to model functionally inhomogeneous materials with the properties of pseudo-elastic-plasticity at large deformations up to 20%, which increases the prediction in the behavior of materials.

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